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CALEB GATTEGNO (1911-1988): A FAMOUS MATHEMATICS EDUCATOR FROM AFRICA?*

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On 11.11.1911, Caleb Gattegno, son of a Spanish merchant, was born in and grew up in Alexandria, Egypt. Later, he lived in Cairo, Egypt; in London and Reading, England; Addis Ababa, Ethiopia; in La Chaux de Fond, Switzerland; and in New York City, USA, but worked all over the world, in all continents. His social concerns and intellectual development as well as his research into mathematics, mathematics education, linguistics, and psychology began in Africa, a continent to which he made several significant contributions.

He was eighth of nine children and, from an early age, had to work for his living. Nevertheless, he always sought to learn and to be an educated person. He committed himself to study, read whatever came his way, particularly mathematics texts, and often earned money by tutoring others. Between the ages of 20 and 25, studying on his own, he took external examinations in Cairo and obtained teaching licenses in chemistry and

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physics (1931), in mathematics (1932), and an advanced studies diploma in mathematics (Diplôme d'Etudes Supérieurs de Mathématiques) in (1936), all from the University of Marseilles, France. He was a self-taught mathematician whose independent studies led him to earn a doctorate in 1937 from Basle University in Switzerland. His thesis title is *Les cas essentiellement géodésiques des équations de Hamilton-Jacobi intégrables par séparation des variables*[1].

In 1932, he founded the Mathematics Seminar, Alexandria's first, modern university-level courses in mathematics. In addition, from then that time until 1936, he was a mathematics teacher at the Lycée Français in Alexandria (Mission Laïque Française). From 1937 to 1945, he founded and directed the Center of Advanced Scientific Studies (Centre d'Etudes Supérieures Scientifiques et Technique) in Cairo, a center for students otherwise not eligible for higher degree, university courses, and in Egyptian journals, published papers on mathematics and education (Tahta, 1989a). Between 1957 and 1958, while working in Ethiopia for a UNESCO mission, whose objective was to find a solution to the problem of illiteracy, he produced textbooks and new teaching materials. Notably, he invented an extraordinary method for teaching the reading and writing of Amharic, by color coding its phonemes and arranging them on charts. It is important to note that Gattegno's functional, pedagogical use of color was influenced by his earlier contact with a Belgian mathematics educator, which will be discussed shortly. Being an accomplished linguist who spoke and worked in several African and European languages, Gattegno developed color-coded charts for most major languages, besides Amharic, including Arabic, Mandarin, English, French, Hindi, Korean, Japanese, Portuguese, and Spanish and developed powerful approaches to the teaching of native language literacy and of foreign languages, which he called *Words in Color* and the *Silent Way*, respectively.[2]

From his early days in Alexandria, Gattegno embarked on a life-long study of human learning, not just in the field of mathematics. An examination of the titles of articles he published while teaching in Cairo reveals that Gattegno sought to understand how social conditions as well as psychology influence learning.[3] His psychological investigations concerned the dynamics of the mind and the role of awareness in learning, *conscience de la conscience*. He pursued these inquiries and, along the way in 1952, earned a second doctorate, this time in psychology from Université de Lille, in France. His thesis was later published in 1954 and again in 1962. [4]

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[1] Translation: The essentially geodesic cases of Hamilton-Jacobi equations integrable by the separation of variables.

[2] A discussion of *Words in Color* and the *Silent Way* lie outside the purpose of this article. Interested readers may consult Gattegno (1976 and 1985a); and from the viewpoint of practitioners of *Words in Color*, see the special issue of *The Science of Education in Questions*, 15, which may be obtained from William Bernhardt, Association for the Science of Education, PO Box 11, Mahopac, NY 10542, USA; or email <bernhardt@postbox.csi.cuny.edu, williambernhardt@hotmail.com>. For *La Science de L'Education en Questions*, 15, in French, write to Une Ecole pour Demain, Cidex 26 bis, Route du Village, 25720 Larnod, France, <http://assoc.orange.fr/une.education.pour.demain/index.htm>.

[3] A comprehensive bibliography of Gattegno's publications prepared by Tahta (1989a) can be found at the following Web site: <http://www.cuisenaire.co.uk/gattegno/bibliog.htm>.

[4] (1954/1962) *Conscience de la conscience*, 2nd edition, Paris: Delachaux et Niestlé.

Just after the end of World War II, Gattegno moved from Africa to Europe. He settled in England, where feeling as an outsider, he organized seminars for displaced people out of a desire to meet and to help them. He also became a lecturer in mathematics at the University of Liverpool and later at the Institute of Education, University of London. In 1948, he completed a Master of Arts in Education at the University of London with a thesis titled, "The Mathematical Definition of Education."

In Europe, Gattegno advanced the notion that mathematics was the birthright of all individuals, emphasizing the role of psychology, epistemology, and manipulative devices. His vision went beyond the post-war economic imperatives of European governments. After the Second World War, European educational systems needed to change to provide more individuals capable of engaging with the technological problems that the Industrial Era had brought into being since the end of the XIX Century. As such, education in general and mathematics teaching in particular needed to change. Addressing this need, Gattegno inspired students and other educators, alike.

Gattegno's leadership was decisive in bring together European mathematics teachers and mathematicians to discuss and analyze how to change the teaching of mathematics so as to increase the number of individuals who succeed in learning mathematics. He was instrumental in establishing several professional organizations: in 1947, L'Ecole Normale Internationale, meetings of educators, professors, and adolescents to investigate the role of education in maintaining world peace, for which he was its unpaid director until 1957; in 1950, the Commission internationale pour l'étude et l'amélioration de l'enseignement des mathématiques (CIEAEM), of which he was its secretary until 1960. In 1952, he participated in founding the Société Belge des Professeurs de Mathématique d'expression française and its journal *Mathematica et Paedagogia*. Also that year, in England, he was a founder of the Association for Teaching Aids in Mathematics (later to become the Association of Teachers of Mathematics) and served as the first editor of its journal, *Mathematics Teaching*.

During his London days and throughout his life, Gattegno persisted with his African praxis: social action combined with research and material development. He was an early English translator of Piaget (1949, *Play, Dreams & Imitations* and 1951, *The Child's Conception of Number*[5]) and influential in spreading knowledge of developmental psychology. After 1949, he acquainted teachers with the silent, unlabelled, and animated geometric films of Jean-Louis Nicolet, a Swiss secondary-school teacher, as well as of his own, and demonstrated how films of the Nicolet genre provide a dynamic approach to geometry in classrooms. He invented geoboards (1952) and incorporated them into his dynamic approach for teaching geometry. In 1955 and again in 1958, based on symposia of the CIEAEM, he edited two books, respectively, *L'Enseignement des Mathématiques* and *Le Matériel pour L'Enseignement des Mathématiques*, among the contributors to these volumes, besides Gattegno himself, were Jean Piaget, Jean-Louis Nicolet, Emma Castelnuovo, Trevor J. Fletcher, Gustave Choquet, and Jean Dieudonné, one of the most

[5] While translating from French to English, *The Child's Conception of Number*, Gattegno (1983) states that he "had opportunities of getting stuck at a number of points" and "suggested to Piaget that Part II of the published study must be different if he wanted the book published in English with [Gattegno] as the responsible translator. [Piaget] yielded…and the English translation differs considerably from the original French" (p. 6).

influential French mathematicians during the 20th century, especially through his association—even identification—with the famous Bourbaki group.

Retrospectively, 1953 can be said to have been a watershed year both for Gattegno and for mathematics education worldwide. In that year, Gattegno became acquainted with the ingenious work of Emile-Georges Cuisenaire (1891-1976), a Belgian schoolmaster, who invented hand-painted *réglettes*, wooden colored rods, and some similarly colored cardboard material (called product cards) to teach his students arithmetic. Gattegno, then a mathematics lecturer at the London Institute of Education and secretary of CIEAEM, was asked to assess the value of Cuisenaire's materials. Cuisenaire found that he achieved something rare: his pupils enjoyed and understood the work they did. With their arithmetical prowess in conformity with the strict European type of syllabus, his students amazed educationalists (Trivett, 1959). In 1952, he published his work in a booklet, titled *Les Nombres en Couleurs*. Nevertheless, for about 23 years, his work and inventions remained almost unknown outside his village of Thuin. However, one year after publishing his book, a providential meeting of this teacher with Gattegno resulted in use of his materials in classrooms throughout the world.

For many years, Gattegno had been a leading figure in the movement to bring improvements to mathematics teaching at the primary and secondary school levels, through employing manipulatives and psychological understanding of human learning. The pedagogical and mathematical potential of Cuisenaire's rods and book impressed Gattegno. As he says, "before I met [Cuisenaire] my own thinking led only to variations of classical themes in the field of number learning" (1960, p. 1). In 1954, Cuisenaire and Gattegno published in England, *Numbers in Colour*, and Gattegno established the first Cuisenaire Company in Wembly, Middlesex, to distribute the Cuisenaire materials. In Gattegno's view, articulated in the preface to Cuisenaire and Gattegno (1954),

> the importance of Cuisenaire's discovery can be formulated in mathematical and psychological terms. Mathematically he has created a material that can exemplify *all* the arithmetical relationships met with in school life, so that we have an aide which is adequate at every stage, when we pass from counting to multiplication, from addition to fractions or to proportion, for we have a *set* of *rods* that can be "structured" in a variety of ways according to the particular relations that is in question.…
>
> Psychologically, the value of Cuisenaire's contribution lies in the fact that by providing a semi-abstract material he has overcome the obstacle of the gap between active and intellectual thought. Our minds are swift when dealing with images and representations but they move slowly in the actual performance of an action. (p. v-vi)

Gattegno realized not only that Cuisenaire's rods physically behave in the way numbers do, providing learners with an algebraic model for the study of mathematics, but also that the rods could be a means for learners to investigate mathematics for themselves. Moreover, he recognized that the colored rods could offer teachers a means for making a lesson a personal investigation of mathematics for each and every learner. On this point, Gattegno (1960) suggests "[s]tudies on the span of attention of children prove that it is dependent on

interest…Perhaps it is verbal teaching that brings about [for children] tiredness and encourages distraction, but with the Cuisenaire rods the children we have watched can, at the age of 5 or 10, keep 'working' for an hour or more and not lose interest or get tired" (p. 8-9).

Significantly and in contradistinction to the then prevailing (and still current) perspective, by 1960, Gattegno based his teaching of arithmetical ideas on the perception of length rather than on the act of counting. In his words, "[i]f we let the children 'have a dialogue' with the rods and if we accept that before they see certain relationships between them they will discover more primitive ones, we shall have eliminated counting as the basis of learning arithmetic" (1960, p. 9). The equivalence or inequality of length of collections of rods is recognized "with the eyes or by touch." For Gattegno this reality coupled with children discussing equivalences or inequalities by using the color name of the rods allows them to base their arithmetical understandings on their action and perception of rod situations and to experience *algebra* before *arithmetic*, which "is much easier to remember than the number [facts]" (Gattegno, 1960, p. 9).

In Gattegno's pedagogical approach, mathematical situations are proposed to learners who, invited to participate actively, become aware, little by little, of the relationships that structure the situations, and at the same time understand better the dynamics of their own mental functionings. Centered on a given, clear, and tangible problem, the situations presented for learners' exploration lead each learner to use their varied mental faculties, to formulate hypotheses and to test them, to develop personal strategies and to arrive at precise and profound understandings. Through those situations, from a perception of what they contain and from actions performed on them, mental representations are created and generalizations made, first verbalized, and finally represented symbolically, in standard mathematical notation.

As learners explore a mathematical situation, teachers have a definite role in Gattegno's pedagogy. After inviting students to work on a given situation, they are help focus students' attention on essential, mathematics aspects of the situation and support mathematical discussion. Teachers should observe how students work on the given situation and be always ready to modify their pedagogical actions, based on what the students are doing, what they are saying, their hesitations and errors, their understandings and misunderstandings, their attitudes and other reactions. Through such constant, formative assessment and by paying attention to each student, teachers can accompany students and guide them step-by-step towards new mathematical insights, concepts, and ways of reasoning. These recommended ways of working in a mathematics classroom reflect his general pedagogical theory that he calls *the subordination of teaching to learning* (see, for example, Gattegno, 1970).

At an international conference—Caleb Gattegno's Science of Education: Ten Years After—in New York City, Shakti Gattegno (1998) summarized seven main tenets of her husband's general pedagogical perspective as follows:

- teaching is guided by learning instead of learning by teaching;
- the subject-matter is the vehicle for learning instead of being the 'target;'
- the learning process, and not the end product, is the focus of attention, with

the understanding that only a well attended process yields good results;

- the mastery of the subject-matter is acknowledged to be the by-product of the learning process, and not the result of teaching;
- mistakes are treated as an integral part of learning and are used as tools for developing the criteria for correctness and for self-correcting by the students;
- teachers relate to the learners' 'strengths' and let the learners take care of their so-called weaknesses; [and]
- teachers are learners, too, learning all along to pose the 'right' challenges and to respond adequately to the learning process of individual learners.

These tenets express Gattegno's belief that education should enhance learners' intellectual independence and autonomy, "to provide students with the means to meet the future" (Gattegno, 1970, p. 1). Considering Cuisenaire's materials, the tenets indicate how Gattegno envisioned their instructional use to improve learners' mathematical experiences in schools.

Gattegno introduced teachers to Cuisenaire's work, applied the use of his rods to teaching higher levels of mathematics, elaborated text materials in several languages for use with Cuisenaire rods, extended the underlying color coding of the rods[6] to a rich variety of rectangular solids and prisms in 1956, and traveled around the world approximately ten times, visiting over forty countries, lecturing on the use of Cuisenaire rods and Gattegno's own epistemological and pedagogical approach to the teaching of mathematics. These activities brought him back to Africa. In 1960, commenting on official attempts to implement his pedagogy and verify his claims, Gattegno (1960) reports that, besides countries in Asia, Europe, and the Americas, his work had attracted the attention of educators in Africa: "South Africa, the Rhodesias [present-day Malawi, Zambia, and Zimbabwe], Central and Eastern African states have been experimenting on a scale which speaks highly of the enthusiasm of local teachers and administrators" (p. 7).[7]

Through his worldwide efforts between 1953 and 1962, traveling to different corners of the planet, Gattegno founded eleven Cuisenaire companies to distribute rods and his own text materials as well as to work with teachers so that they could learn to implement his pedagogical approach. Through the educational activities of the Cuisenaire companies as well as his lectures, workshops, and books, he helped many thousands of teachers all over the world realize that Cuisenaire rods offer insights into algebraic, spatial relations underlying arithmetic and that the rods model the *algebra* of the rational number system or,

[6] Cuisenaire imbued the rods with functional and interesting correspondences between color and length. Rods of the same color are the same length and, conversely, rods of the same length have the same color. A length that is the double of another is also darker in tone. The smallest rod is a neutral color, usually referred to as "white." Rods that are equivalent in length to two, four, and eight white rods have pigments with an affinity to the red family (purple and brown). Rods equivalent in length to 3, 6, and 9 white rods are colored respectively light green, dark green, and blue; while rods equivalent to 5 and 10 white rods are yellow and orange. The rod whose length has the distinction of being relatively prime to rods of all other lengths in the set is colored black.

[7] At the London Institute of Education, Gattegno taught a diversity of African students from the then British colonies. After contact with his ideas, some students would return home and attempted to implement Gattegno's pedagogical ideas. Such is how experiments occurred in places like South Africa. Though invited, he refused to visit the Republic of South Africa, owing to its then policy of Apartheid.

more abstractly, a *commutative field*. He advocated that teachers allow students to encounter *algebra before arithmetic* as they explore and discuss their *actions* on and *perceptions* of rod situations, and write statements about them, using the rods color names. The rods are objects embodying implicit mathematical relations, which are made salient through learners' awareness in dialogue with the materials and others (Gattegno, 1960, 1963, 1970, 1974, 1975, 1987, 1988b).

By 1965, Gattegno had left London for New York City, where he founded Schools for the Future and later established an educational laboratory, Educational Solutions, Inc.[8] Through both organizations and with the assistance of teacher educators who worked with him,[9] he disseminated his educational ideas, distributed his pedagogical materials, trained thousands of teachers, and worked in hundreds of schools all over the USA, including, in particular, special projects with teachers and schools in the African American communities of Harlem and the South Bronx of New York City as well as in southern cities of the United States, many of which were just emerging from decades of strict racial segregation. Furthermore, from his New York base, he continued to travel worldwide—including to Brazil, having been invited by the mathematician and educator Ubiratan D'Ambrosio—and maintained a rigorous schedule for giving seminars and workshops, for developing new materials, for researching, and for writing.

Gattegno's intellectual production was prolific and incorporated advances in technology. He wrote many important articles and books on mathematics teaching such as *For the Teaching of Mathematics* (1963, three volumes),[10] Functioning as a Mathematician (1967), *The Common Sense of Teaching Mathematics* (1974), The Foundations of Geometry (1980), Curriculum and Epistemology (1984a), Curriculum and Epistemology II (1984b), and Reflections on Forty Years of Work on Mathematics Teaching (1988c). He developed films for teaching trigonometry;[11] computer-animated films for teaching geometry, synthesizing and extending some themes of Nicolet's films[12] as well as initiating some of his own;[13]. From 1980 to 1982, with financial support from the National Science Foundation in the United States and using then newly available desktop digital technology, Apple IIe microcomputers, he created innovative software to teach numeration, addition, and subtraction, incorporating ideas of equivalence, complementarity, and transformations.[14]

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[8] Readers interested in obtaining Gattegno's publications and educational products should address inquires to Educational Solutions, Inc., 99 University Place, New York, NY 10003-4555, USA; or telephone 212.674.2988 <http://members.aol.com/edusol99/index.htm>.

[9] Among the mathematics educators who worked with Gattegno during his New York days are Nancy Austin, Sandy Dawson, Martin R. Hoffman, Marilyn Maye, Arthur B. Powell, Stephen Shuller, Dick Tahta, John V. Trivett, and David H. Wheeler,

[10] The international journal, *For the Learning of Mathematics,* is probably so named in homage to Gattegno. It was launched in 1980 by David H. Wheeler five years after he immigrated to Canada, having worked from 1973 to 1975 with Gattegno in New York City.

[11] Produced in the 1960s, *Folklore of Mathematics*, and available from Educational Solutions.

[12] 1981, *Animated Geometry*, New York: Educational Solutions.

[13] 1979, *Foundations of Geometry*, New York: Educational Solutions.

[14] 1982, *Visible and Tangible Mathematics*, New York: Educational Solutions.

Importantly, Gattegno posited perception and action as bases for mathematical thought and mathematics as the study of the dynamics of relationships *per se*. He developed classroom materials and an pedagogical approach that invite learners to engage their powers of perception and action to mathematize situations and, thereby, become aware of their own mathematical ideas as they make explicit their awareness of relationships implicit in these situations.[15] He argued that teachers should "use the time in class to make students mathematize situations and discover how many chapters of mathematics can be deduced, induced, from a minimum of givens" (1984b, p. 21). Referring to this latter notion, he coined the phrase *a lot from a little*, which as Gattegno (1984b) explained means "'[g]ive students only what they cannot reasonably find by themselves and let them do the rest'" (p. 21). Furthermore, except for "what they cannot reasonably find by themselves" such as labels and terms, Gattegno's pedagogical approach does not require students to memorize facts and algorithms. Instead, he understood that "mathematization is not only accompanied by the joy of discovery but also by functional retention" (1988b, p. 132). These annotations point to a few elements of his epistemology. However, they barely scratch the surface of his thinking. A succinct, thought-provoking summary of his epistemological theory contained in Gattegno (1987) and of life's work in mathematics education is found in his book, published posthumously, *The Science of Education, Part II: The Awareness of Mathematization* (1988b).

Moreover, Gattegno engaged individuals in his seminars in particular ways. He practiced his pedagogical theory of subordinating the teaching of his particular theories to the learning of seminar participants. As Lopes (2006) notes "Gattegno, nesses seminários, suscitava o trabalho de grupo, favorecia as trocas para confrontar concepções diferentes e tornava possível o conhecimento e a análise de pesquisas ou experiências de ensino dos participantes, sem discriminação entre pesquisadores renomados e simples professores"[16] (p. 15). He was both famous and infamous for his seminars. He often would insist that seminar participants rely on their analysis of their experience rather than on the authority of received knowledge.

Who was Caleb Gattegno? This question is difficult, even impossible to answer especially since here we focused on only a few aspects of the man. Nevertheless, we can say that, though he descended from a family of Sephardic Jews,[17] lived in Egypt, once held a Spanish passport,[18] and eventually gained British citizenship, Gattegno bypassed nationality, freed himself of labels and flags (Tahta, 1997), and became a citizen of the world. He was an autodidact, a scientist, an inventor, a teacher, and a student of human learning. He not only was an educator of international proportions who created a number

[15] Many practitioners have written about their implementation of Gattegno's theoretical ideas and pedagogical approach in mathematics, but a list of such works would be far too numerous to mention here. Nevertheless, six such articles and a doctoral thesis by this author are Powell (1986, 1993, 1995, and 2003), Powell and Hoffman (1991), Powell and Maher (2003), and Powell and Ramnauth (1992),

[16] Translation: In these seminaries, Gattegno would promote group work, would prefer for participants to exchange and challenge different conceptions, and would make possible knowledge and analysis of their research or teaching experiences, without discrimination between famous researchers and simple professors.

[17] It is most probably that Gattegno's ancestors immigrated to Egypt due to the ruthless expulsion of Muslims and Jews from Spain during its Inquisition (1478-1834).

[18] Gattegno renounced his Spanish citizenship in protest over the fascism of Franco's reign in Spain.

of important, innovative techniques for the teaching and learning of languages and mathematics but also made seminal contributions to understanding the learning process, at all ages (Tahta, 1989b). He published more than fifty books and countless other writings on his epistemological, psychological,[19] and pedagogical research, including his theories concerning the teaching of mathematics, reading, writing, and languages. He believed in and respected the powers that every person possesses for learning any discipline. In 1985, referring to mathematics education in the final paragraph of the preface to *Aperçu historique sur la Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques*,[20] he states that "personne ne devrait étre privé de la joie de la découverte mathématique que nous savons étre á la portée de tous parce qu'elle a été á notre portée"[21] (Gattegno, ca. 1985). For Gattegno, this statement is neither gratuitous nor reflective of merely a romantic, egalitarian sentiment of a socialist discourse. In his writings and seminars, he forcefully insisted on this statement since he understood that, in every cultural group, each person independently and autonomously teaches himself or herself so much during the first years of life, not the least of which is to speak. He presented evidence for his claims in many of his books such as in *What We Owe Children* (1970), *In the Beginning There Were No Words: The Universe of Babies* (1973), *The Mind Teaches the Brain* (1975/1988), and *The science of education, Part I: Theoretical Considerations* (1987).

Caleb Gattegno died from cancer on 28.7.1988 in Paris, two weeks after conducting a seminar, "*Le mystère de la communication*," near Grenoble, France.

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[19] In a brief statement indicating the focus of his investigations in psychology and their contribution to the field until then, Gattegno (1979) summarizes it as follows:

> In my own work (which I located within psychology) I studied learning as a conscious act of a self endowed with many attributes such as awareness, will, intelligence, discrimination, retention, perception, imaging and so on. I became convinced that psychology could be defined as the science of time, the time we consume for experiencing. Looking at what we did with our time from conception on helped me shed new light on several phenomena which either had been poorly understood until then or had been left out altogether. In particular, my studies of learning yielded not only what Cognitive psychologists hoped to find in their own studies but allowed me to offer a technology for education that proved me closer to knowing what knowing is than was possible in the existing laboratories. (p. 8)

Here he uses technically terms such as *self*, *awareness*, and *knowing*. To expose his meanings would require more space than allotted. Fortunately, however, he discusses his technical terms in many of his publications (see, for example, Gattegno, 1988a). He gave his "technology for education" material expression in the materials he developed for the teaching and learning of reading, writing, languages, and mathematics.

[20] Translation: A Brief History of the International Commission for the Study and Improvement of the Teaching of Mathematics.

[21] Translation (by Dick Tahta (1997, personal communication): "no-one should be deprived of the joy of mathematical discovery that we know to be within everyone's reach because it has been within ours."

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