Revista Brasileira de História da Matemática - Vol. 24, nº48 - pp. 75–89
Publicação Oficial da Sociedade Brasileira de História da Matemática
ISSN 1519-955X

THE SCIENTIFIC CONTRIBUTION OF LUIGI FANTAPPIÈ TO THE FORMATION OF THE BRAZILIAN MATHEMATICAL COMMUNITY

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(aceito para publicação em abril de 2023)

Abstract

The purpose of this paper is to emphasize the pioneering scientific contribution of the Italian analyst Luigi Fantappiè, from 1934 onwards to the beginning of the process of formation of the Brazilian mathematical community and, his influence on the elaboration of the first two theses of Doctorate in Sciences (Mathematics) which were defended at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo in 1942 and 1944. We briefly comment on aspects of L. Fantappiè's scientific work in the theory of analytical functionals with contributions to the development of Mathematical Analysis in the 1930s, 1940s, and 1950s.

Keywords: Luigi Fantappiè; Analytical Functionals; FFCL-USP; Cândido Lima da Silva Dias; Omar Catunda; Brazilian Mathematics Community.

[A CONTRIBUIÇÃO CIENTÍFICA DE LUIGI FANTAPPIÈ PARA A FORMAÇÃO DA COMUNIDADE MATEMÁTICA BRASILEIRA]

Resumo

O objetivo deste artigo é ressaltar a contribuição cientifica pioneira do analista italiano Luigi Fantappiè, a partir de 1934, para o início do processo de formação da comunidade matemática brasileira e, sua influência para a elaboração das duas primeiras teses de Doutorado em Ciências (Matemática) que foram defendidas na Faculdade de Filosofia, Ciências e Letras da Universidade de São Paulo em 1942 e em 1944. Comentamos brevemente aspectos do trabalho científico de L. Fantappiè na teoria dos funcionais analíticos com contribuições para o desenvolvimento da Análise Matemática dos anos 1930, 1940 e 1950.

Palavras-chave: Luigi Fantappiè; Funcionais Analíticos; FFCL-USP; Cândido Lima da Silva Dias; Omar Catunda; Comunidade Matemática Brasileira.

Introduction

The purpose of this article is to highlight the valuable scientific contribution made by the Italian analyst Luigi Fantappiè (1901-1956) to the formation process of the Brazilian mathematical community. The contribution was in organizing and teaching disciplines (with the inclusion in the program of fields of current themes for the time) of the graduation course Bachelor in Mathematics offered by the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo (FFCL-USP), and mainly, through its orientation to young professors from the Department of Mathematics at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo.

Luigi Fantappiè arrived at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo in July 1934, hired as a Visiting Professor by mathematical engineer Prof. Dr. Theodoro A. Ramos (1895-1937)[1], the first Director of the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo, to govern the disciplines of Mathematical Analysis and Geometry of the Bachelor's Degree in Mathematics. This was a wise choice by the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo by the Creative Committee.

The Faculty of Philosophy, Sciences, and Letters of the University of São Paulo was the first institution to operate in Brazil as an official institute of high culture, not a professional one. The government of the state of São Paulo assured the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo of absolutely original guidance under the wishes of scholars and researchers. Its activities began on March 11, 1934.

Contextual Elements

1 - General information

Prof. L. Fantappiè specialized in Functional Analysis. He was the creator of the Theory of Analytical Functionals, an important subarea of Mathematical Analysis at the time. L. Fantappiè published various articles from the 1930s. He considered in his ideas the space of ultra-regular functions on the complex sphere equipped with an appropriate topology.

In his work on the Theory of Analytical Functionals, Prof. L. Fantappiè defined an analytic curve as an ultra-regular function y(t, α) depending analytically on the complex parameter α, (FANTAPPIÈ, 1943).

So, L. Fantappiè stated that a functional F (see below the idea of functional that was considered at the time) was analytic if the following conditions were met:

- F was defined on an open subset of the ultra-regular functions.
- F(y) = F(y_0) if y is an extension of y_0.
- For an analytic curve y(t, α), the function F[y(t, α)] = f (α) i analytic and regular at those points α for which y(t, α) belongs to the domain of F.

[1] Francesco Severi (1879–1961) recommended L. Fantappiè to Theodoro A. Ramos. F. Severi was a member of the Italian school of Algebraic Geometry.

In developing his ideas, Prof. L. Fantappiè used the function y(t, α) $= \frac{1}{(\alpha-t)}$ which is a special analytic curve which he used to define the indicatrix:

$$F\left(\frac{1}{\alpha-t}\right) = \gamma(\alpha) \qquad (**)$$

of a functional F.

The indicatrix ($**$) was very important in this context, it helped Prof. L. Fantappiè prove the following result or theorem:

$$F[y(t)] = \frac{1}{2\pi i}\int_C \gamma(t)y(t)dt, \qquad (***)$$

where γ is the indicatrix of the functional F and C is an appropriately chosen contour. Operator ($***$) of a functional as a Cauchy's integral formula is the main result in Fantappiè's theory of linear analytic functionals. Using it, L. Fantappiè gave a new basis for operational calculus. (FANTAPPIÈ, 1943); (CATUNDA, 1944); (DIAS, 1942).

L. Fantappiè's theory of analytical functionals is an example of applications of the theory of functionals to concrete Mathematical Analysis. We conjecture that L. Schwartz, between 1947 and 1950, used the ideas of L. Fantappiè when creating the theory of distributions[2], which extends the concept of function, even though L. Schwartz was interested in investigating highly irregular functions. The paper (FANTAPPIÈ, 1930) was quoted by Laurent Schwartz in his book, (SCHWARTZ, 1950; 1951).

The work developed by L. Fantappiè on the Theory of Analytical Functionals was useful for the beginning of the creation of the Theory of Topological Vector Spaces. It is known that the general theory of Topological Vector Spaces emerged in the period between the 1920s and 1930s. Many mathematicians contributed to its development, for example, D. Hilbert (1862–1943) with the theory of Hilbert spaces. At the beginning of the 20th century, new theories were added to Topological Vector Spaces, such as the theory of normed spaces; the general definition of normed spaces was given between the years 1920 and 1922 by the mathematicians S. Banach (1892–1945), H. Hahn (1879–1934) and E. Helly (1884–1943).

In this context, the general idea of a functional emerged in the last decades of the 19th century. Namely, a function with numerical values defined in a set whose elements are numerical functions of one or more real variables. This idea was related to the calculus of variations and the theory of integral equations. Ideas that Italian mathematicians like V. Volterra worked on. We remember that L. Fantappiè was a pupil of V. Volterra; therefore, he was immersed in this context of the Italian school at the beginning of the 20th century. We also remember that, according to (BOURBAKI, 1994, p. 213), in the early days of the theory of topological vector spaces, J. Hadamard started the theory of topological duality in 1903, working with the most general continuous linear functionals on the space $\mathcal{C}$(I) of

[2] According to the mathematical literature, S. Sobolev (1908-1989) invented distributions, but it was L. Schwartz (1915-2002) who created the theory of distributions. See (SCHWARTZ, 1950, 1951).

numerical functions on a compact interval I, which is equipped with a topology of uniform convergence. He characterized them as limits of sequences of integrals:

$$x \longrightarrow \int_I k_n(t)x(t)dt.$$

In the early days of modern integration theory, on the issue of integration in locally compact spaces, F. Riesz (1880–1956) solved in 1909 a problem proposed by J. Hadamard, demonstrating that Stieltjes integrals $f \longrightarrow \int_a^b f d\varphi$ a linear functionals more general continue the space $\mathcal{C}$(I) of continuous numerical functions over the interval I = [a, b] with $\mathcal{C}$(I) the topology of uniform convergence. (RIESZ, 1911). After the publication of this result by Riesz, several generalizations immediately emerged.

Through his Theory of Analytic Functionals, L. Fantappiè contributed to the development of Locally Convex Spaces, a definition given in 1935 by J. von Neumann (1903–1957). The growth of these spaces occurred thanks to the development, from 1930 to 1940, of the fundamental notions of General Topology and the boost in the 1930s of works motivated by the new possibilities of applying mathematical analysis in an area in which S. Banach's theory proved inoperative. As a byproduct of the development of Locally Convex Spaces, we can mention:

- The Theory of Succession Spaces, developed by G. Göthe (1905–1989) and O. Toeplitz (1881–1940).
- The Theory of Distributions, developed by L. Schwartz.

In the latter, the modern Theory of Locally Convex Spaces found a fertile field of applications.

2 - Activities of L. Fantappiè at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo and consequences

With the information mentioned above, we want to highlight the environment for the creation of new subareas of Mathematics that took place in Europe in the 19th and early 20th centuries, in which L. Fantappiè was formed. Thus, he transmitted to his students, from 1934 onwards, at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo, what was current in some of the subareas of Mathematics at the time. L. Fantappiè through his teaching in the course Bachelor of Mathematics from the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo, opened the door for talented students to see what the European mathematics community was created in: Foundations of Mathematics; Logic; Set Theory; Combinatorial Analysis; Evolution of Algebra; Linear Algebra and Multilinear Algebra; Polynomial and Commutative Fields; Commutative Algebra; Algebraic Number Theory (we remember that the importance of the work of J. F. C. Gauss *Disquisitiones Arithmeticae* for Number Theory in the 19th and 20th centuries); Non-Commutative Algebra; Topological Spaces; Real Numbers; n Dimensional Spaces; Complex Numbers; Metric Spaces; Infinitesimal Calculus; Function Spaces; Integration in Local Spaces; Lie Groups and Lie Algebras; Groups Generated by Reflections; Root Systems; Complex Analysis;

Functional Spaces; Topological Vector Spaces; Algebraic Geometry; Tensor Analysis (at the time called Absolute Calculus), etc.

The Commission that created the Faculty of Philosophy, Sciences, and Letters at the University of São Paulo wanted the undergraduate courses to be offered by the institution to be of a good standard. As in Brazil, there were no qualified teachers for this project, he commissioned Prof. Dr. Theodoro A. Ramos to hire qualified teachers in Europe who wanted to work on this project. With the help of European colleagues, Theodoro A. Ramos visited several countries and, in Italy when visiting mathematicians and physicists, he was recommended to talk with the analyst L. Fantappiè and with the experimental physicist Gleb Wataghin[3]; this one, for the graduation course in Physics. Both accepted the invitation to work at the newly created Faculty of Philosophy, Sciences, and Letters of the University of São Paulo.

At the time, the Bachelor of Mathematics degree offered by the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo was a three-year program, like nowadays, which Prof. L. Fantappiè considered a short time. However, the solid mathematical training of Prof. L. Fantappiè and his enthusiasm for creating and teaching excellent undergraduate courses contaminated the young professors and students from the undergraduate program in Mathematics. From that moment on, in 1934, the process of forming the Brazilian mathematical community began, starting from São Paulo city.

The main pupils of Prof. Dr. L. Fantappiè at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo were: Omar Catunda, Cândido Lima da Silva Dias, and Benedito Castrucci. The first two were chosen assistants of Prof. L. Fantappiè by Prof. Dr. Theodoro A. Ramos. Both began to teach classes in basic disciplines indicated by L. Fantappiè and also began to study the Theory of Analytical Functionals. This fact encouraged them to subsequently defend their first Doctorate theses in Science (Mathematics) at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo[4].

When organizing the disciplines of the Bachelor of Mathematics program, Prof. L. Fantappiè taught in the 2nd year of the program the courses Mathematical Analysis, and Elements of the Theory of Analytical Functions. A novelty in the higher education of Mathematics in Brazil at the time. The courses involved studying functions of a real variable analytic function known as special functions.

For the reader unfamiliar with these functions, we will provide some definitions. But, first, we inform you that functions that admit developments in power series, such as the exponential function, the logarithm function, the sine function, and the cosine function, form an important class of functions called analytic functions.

Definition 1. The function f: (a, b) $\longrightarrow \mathbb{R}$ is said to be analytic at a point x_0 on the interval (a, b), when f is representable by a power series:

$$f(x) = a_0 + a_1(x- x_0) + a_2(x- x_0)^2 + \cdots$$

[3] Gleb Wataghin was a disciple of the theoretical physicist Enrico Fermi (1901- 1954).

[4] Subsequently, both defended other theses for competitions for the provision of Chairs at Faculty of Philosophy, Sciences, and Letters of the University of São Paulo.

uniformly convergent on $(x_0 - r,\ x_0 + r) \subset$ (a, b). Where ℝ is the set of real numbers.

Definition 2. The function f: (a, b) ⟶ ℝ is said to be analytic at (a, b) when the function f is analytic at every point of (a, b).

Definition 3. If Ω is an open set in ℂ and f: Ω ⟶ ℂ then f is differentiable at a point a in Ω if

$$f'(a) = \lim_{h \to o} \frac{f(a+h) - f(a)}{h}$$

exist. ℂ is the set of complex numbers. If f is differentiable at each point a of Ω, f is differentiable on Ω.

The following proposition is proved.

Proposition. If f: Ω ⟶ ℂ is differentiable at a point a in Ω then f is continuous at a.

Definition 4. A function f: Ω ⟶ ℂ is analytic if f is continuously differentiable on Ω.

In the mathematical literature, we find the first concept of Analytical Functional Space in 1901 (PINCHERLE; AMALDI, 1901). In that article, the authors consider the following.

Given r ∈ $[0, \infty)$, let $\mathcal{S}$[r] be the set of all power series of z ∈ ℂ whose radius of convergence is greater than r. Given two power series:

$$\sum_{n=0}^{\infty} a_n z^n \ and \ \sum_{n=0}^{\infty} b_n z^n;$$

Let us consider its sum: $\sum_{n=0}^{\infty} (a_n + b_n) z^n$,

and the product by a complex number k:

$$\sum_{n=0}^{\infty} (k a_n) z^n.$$

Equipped with these operations, the set $\mathcal{S}[r]$ is a complex vector space.

However, the authors mentioned above could not introduce the concept of limit into these functional spaces. This lack of the concept of limit was a problem that persisted in studies of spaces of analytic functions.

When studying analytic functions in the context of Functional Analysis, Prof. L. Fantappiè noticed this flaw in the theory. He tried to remedy it by introducing the concept of analytical function depending analytically on a parameter, which allowed him to build the main results of the Theory of Analytical Functionals.

Definition 3. An analytic function φ(z, α) is said to depend analytically on the parameter α, when φ(z, α) is the analytic function of the two variables z and α.

According to Prof. L. Fantappiè, when α varies over a domain D, the function φ(z, α) describes an analytical line in the functional space $\mathcal{S}$.

Prof. L. Fantappiè encountered great difficulties when expanding the concept of analytical functional space. He proposed considering locally analytic functions as elements of an analytical functional space to resolve these difficulties.

The theory of analytic functionals, developed by L. Fantappiè, was based on the concept of linear functional region, using the concepts of sum and product of functions.

However, L. Fantappiè's work on the concept of linear functional region was met with skepticism by the mathematical community, as illustrated by the criticism made by the Portuguese mathematician José Sebastião e Silva. We reproduce, in free translation, the text of the Portuguese mathematician.

> "*Let us not forget, however, that the sum or product of two locally analytic functions must still be a locally analytic function which, as such, cannot fail to have a certain domain of existence. This could be, for example, the intersection of the domains of the given functions; but it is easy to see that, however one tries to choose this domain of existence, the set (ℂ) can never be an additive group – nor can it therefore be a vector space. […]. Another difficulty, more substantial than the previous one, arises with the problem of defining a suitable topological structure in the set of locally analytic functions. The concept of "neighborhood", introduced by Fantappiè in his functional space, does not seem susceptible of fruitful use: the entire theory of analytical functionals can be developed, in what is essential, without the intervention of such a concept. In particular, it does not allow us to define an adequate concept of "limit" for sequences of functions. And that is precisely where the origin of its low fertility must lie.* […]". (SILVA, 1950, p. 3, 4).

In the 3rd year of the Bachelor of Mathematics undergraduate program at the Faculty of Philosophy, Science, and Letters of the University of São Paulo, Prof. L. Fantappiè taught the course Functional Analytical Theory. This fact indicates the quality of the offered undergraduate program, as the study of topics dedicated to current scientific research is usually relegated to a Ph.D. program.

With the continuation of the 2nd World War, the State of São Paulo government rescinded, in 1939, the employment contract that Prof. L. Fantappiè had with the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo. Prof. L. Fantappiè returned to Italy in 1939[5], and Prof. Omar Catunda was designated responsible for the Mathematical Analysis chair.

In 1942, the government of the state of São Paulo made postgraduate studies official at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo, through State Decree-Law No. 12,511, of January 21, 1942, which reorganized the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo.

The first two doctoral theses in Sciences (Mathematics) defended at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo were both written by pupils

[5] When Italy allied with Germany in 1939, B. Mussolini (1883–1945) urged Italians living abroad to return to Italy. The episode in L. Fantappiè's life that we believe tainted his academic life was his choice to embrace, follow and disseminate the fascist political ideology, characterized by dictatorial power and repression of the opposition, which was adopted by the Italian government in the 1930s and part of the 1940s.

of Prof. L. Fantappiè and addressed topics from the Theory of Analytical Functionals, which had been created by L. Fantappiè. The first of these theses is:

> • Prof. Cândido Lima da Silva Dias, who obtained the title of Doctor of Science (Mathematics) from the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo on November 11, 1942, by defending the thesis entitled "Sobre a Regularidade dos Funcionais Definidos no Campo das Funções Localmente Analíticas" (About the Regularity of Defined Functionals in the Field of Locally Analytical Functions). Subarea: Mathematical Analysis. Advisor: Prof. Omar Catunda[6].

This was the first Ph.D. thesis in Science (Mathematics) defended at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo. In his thesis, according to information that we have about this work, the author addressed the concept and foundations of the theory of linear analytical functionals. This Thesis was influenced by the scientific teachings of Prof. Dr. L. Fantappiè. Unfortunately, this thesis does not exist, neither in PDF nor in printed format, in the archives of USP's libraries. There is only information that the thesis was presented and defended on December 11, 1942. In this same line of work, in 1943, Prof. Dr. Cândido Lima da Silva Dias published an article in the Annals of the Brazilian Academy of Sciences, (DIAS, 1943). The text paper dealt with Analytical Functionals, a fact that shows us the scientific influence exerted by L. Fantappiè on young professors of the Department of Mathematics at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo. In this article, the author discusses the concept and foundations of the Theory of Analytical Functionals developed by Prof. Dr. L. Fantappiè and introduces a concept of regularity for functionals defined in the space of local analytic functions. As we couldn't review the Doctoral thesis of Prof. Cândido Lima da Silva Dias, which was defended in 1942 at the Faculty of Philosophy, Sciences and Letters of the University of São Paulo, we conjecture that this paper is a summary of his Doctoral thesis.

By particularizing the concept of linear functionals, he showed in the continuation of the article that these are linear analytics in the sense of L. Fantappiè.

In this paper, Prof. Cândido Lima da Silva Dias addresses and develops the following topics:

1 - Functional definition field. Surroundings of an analytic function. Functional regularity.
2 - Functional additive. Complex additivity. Homogeneity. Linear functionality.
3- Linear functionals $F_t[y(t)]$ are analytic in Fantappiè's sense.
4 - Demonstration of the fundamental formula.

The author informed us in the article that the following property was demonstrated by Prof. L. Fantappiè using the fundamental formula of linear analytic functionals:

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[6] . At the time, Prof. Omar Catunda was not a doctor, but he was the interim head of the Chair of Mathematical Analysis at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo. This legal status allowed him to supervise doctoral theses.

> "*A linear functional F applied to a series of functions* $\sum_{n=1}^{\infty} ny_n(t)$ *uniformly convergent in a region* $\bar{A}$ *, which contains in its interior all the points of the set A in which the indicatrix is defined, produces as a result the sum of the series of values that the functional assumes for each of the terms* $y_n(t)$ *of the series*". (DIAS, 1943, p. 1).

Prof. Dr. Cândido Lima da Silva Dias called this property: "continuity of linear analytical functionals".

On page 6 of the article, Prof. Cândido Lima da Silva Dias says that the demonstration of the fundamental formula of linear functionals is a consequence of the integral formula of Augustin Louis Cauchy (1789–1857) for functions of complex variables, and of the regularity condition of the functional that he introduced earlier.

Let us recall the Cauchy integral formula, which is given like this.

Let f: D ⟶ ℂ is an analytic function defined over a simply connected region D in the complex plane ℂ. Then,

$$f(p) = \frac{1}{2\pi i} \int_K \frac{f(z)}{z - p}\, dz,$$

where K is a closed path entirely contained in D and p is a point inside K.

With the Cauchy integral formula, the first derivative of an analytic function can be obtained. It is known that in the study of functions of complex variables, if a function f has a continuous first derivative, then it also has continuous derivatives of all orders. To obtain these derivatives, just use the Cauchy integral formula[7]. This implication is not true when working with functions of real variables.

In the continuation of the article quoted above, Prof. Cândido Lima da Silva Dias makes remarks; and, using Cauchy's integral formula, he demonstrates the intended formula, which is as follows:

$$F_t[y(t)] = \frac{1}{2\pi i} \int_{C'} y(\tau) u(\tau) d\tau.$$

Where C' is the contour of the curve C, the curve that surrounds the region R_1 and the τ are points on C'. (DIAS, 1943, pp. 6-9).

The second doctoral thesis in Sciences (Mathematics) defended at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo were.

- Prof. Omar Catunda, who obtained the title of Doctor of Sciences (Mathematics) on March 9, 1944, by defending the thesis entitled "Sobre os Fundamentos da Teoria dos Funcionais Analíticos", (About the Fundamentals of the Theory of Analytical Functionals), in a competition for the Chair of Mathematical Analysis, in the

[7] For a demonstration of Cauchy's integral formula, in a didactic, simple and elegant way, see (MEDEIROS, 1972, pp.115, 116).

Mathematics Department of the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo.

Prof. Omar Catunda decided to defend the thesis in a contest to fill the Chair of Mathematical Analysis at the Department of Mathematics at defended Faculty of Philosophy, Sciences, and Letters of the University of São Paulo. This Chair was vacant. We remember that, at this time and for many years afterward, the public contest for the provision of a Chair at a Brazilian University granted the approved candidate the title of Doctor (for the case of USP, see State Decree-Law No. 12,511, of January 21, 1942).

In this work, Prof. Omar Catunda systematically studied the fundamental part of the Theory of Analytical Functionals, which was created by Prof. Dr. Luigi Fantappiè. When this specialty of Mathematical Analysis was created, it was successfully applied in the theory of functions of matrices and operators, in the resolution of certain types of differential equations, and in the foundation of symbolic calculus, among other areas of Mathematics.

However, the central problems of this new theory began to emerge already in its foundations. For example, scholars in this new subfield of Mathematical Analysis, Analytical Functional Spaces, were unable to introduce the concept of limits in functional spaces in the real field. Mathematical Analysis results, when applied to Functional Analytical Spaces, presented major problems, as the definitions hitherto known and introduced by Fréchet (M. Fréchet (1878–1973)) were hardly applicable to these spaces. Another problem with this new theory was the difficulties that arose when analytic functions in the sense of Weierstrass (K. Weierstrass (1815–1897)) were considered as elements of the functional.

Remember that this problem was addressed above. This fact, according to Prof. Omar Catunda, simplified the studies on continuity in lines and in analytic varieties, but introduced a complication, because for locally analytic functions it would be necessary to consider as a point a complex formed by the function and by the region in which the function is defined.

In this way, according to Prof. Omar Catunda, the same analytic function in the strict sense, gives rise to an infinity of points, which are obtained by taking all possible regions contained in its field of existence. As a result, the functional space becomes extraordinarily complicated, making its topological study difficult.

To overcome this difficulty, in his thesis, Prof. Omar Catunda introduced the concept of point (f, R) with the notions of distinct and essentially distinct points, necessary for the study of certain topological relationships. The points thus defined, completed with the concept of surroundings (T, σ), form a space with very different properties from most of the functional spaces studied.

In Chapter III of his thesis, Prof. Omar Catunda introduced the concept of continuous functional by adapting the definition of continuity for successions to the general concept of continuity of a functional defined in a surroundings space. This suggestion was made, according to the author, by Prof. Cândido Lima da Silva Dias.

On page 3 of his work, Prof. Omar Catunda informs us the following:

> "*Among the continuous functionals defined in the analytic functional space, the simplest are the linear ones, which can also be defined by the condition*

> *of "complex additivity". For these we establish Fantappiè's fundamental formula, modified in such a way as to eliminate the restriction required in his theory, referring to the regularity of a function at infinity. The proof is based on Prof. Cândido Dias, replacing however the proof by succession with the proof by continuity. With which Heine's theorem is avoided, and therefore Zermelo's postulate. From this formula, in turn, one deduces the property that Prof. Fantappiè saw it as defining the analytic functionals*". (CATUNDA, 1944, p. 3)

Heine's theorem (E. Heine (1821–1881)) referred to by Prof. Omar Catunda is the following:

> **Heine's theorem**. The necessary and sufficient condition for a function f(x), defined in a metric space, to be continuous at a point x, is that for every sequence $\{x_n\}$ convergent to x, it has $lim f(x_n) = f(x)$.

Finally, Prof. Omar Catunda informs us in his thesis that he had stopped studying, due to lack of time, some important problems such as the following: the application of the theory of analytic functionals, by processes of passing to the limit, to the study of defined functionals for continuous real functions, or of some Baire (René-Louis Baire (1874–1932)) class defined on a real interval [a, b].

On page 48 of Chapter III of his thesis, Prof. Omar Catunda in § 4, aims to obtain a fundamental formula that has the Fantappiè formula as a particular case, but that is applicable without the restriction on regularity at infinity. Then, he proceeds to deduce, from the conditions of continuity, linearity and interchangeability with the factor i, for a functional F defined in a linear functional region $\mathcal{H}$, the sought formula.

For that purpose, Prof. Omar Catunda starts using the modified Cauchy integral formula:

$$f(z) = \frac{1}{2\pi i} \int_C \frac{f(t)(z-\lambda)}{(t-\lambda)(t-z)} dt;$$

where C is a regular contour enclosing a region R_f where f(z) is analytic, and over the region f(z) is continuous, with z an internal point of this region and λ an external point of the region. After technical development that we will omit, Prof. Omar Catunda gets:

$$F[f(z)] = \frac{1}{2\pi i} \int_C f(t) u_\lambda(t) dt, \qquad (*)$$

which is the fundamental formula of the theory of linear analytic functionals.

Prof. Omar Catunda informs us on page 50 of his thesis that from the formula (∗) the following important consequence can be deduced:

> "*If the point f(z) describes an analytic line (f(z, α), R(α)) contained in the functional region $\mathcal{H}$, then the corresponding value of the functional is an*

> *analytic function of α in the entire region Ω in which this parameter varies*". (CATUNDA, 1944, p. 50)

Next, he explains how to prove this consequence and gives the technical proof that we will omit.

On page 57 of the thesis and in the continuation of §5: “Changes of Variables in Linear Functionals”, Prof. Omar Catunda demonstrated the following Theorem referring to analytic functions. According to the author, this is a theorem of great importance and not known by the mathematical community.

> **Theorem**. If a regular and monodromic function ϑ(t) at all points of a simple regular contour C is such that for any analytic function g(t) on that contour and on the inner part R, we have:
>
> $$\int_{C'} g(t)\vartheta(t)dt = 0,$$
>
> Then the function ϑ(t) can be extended by a monogeneous function over the entire region R. (CATUNDA,1944, p. 57).

To demonstrate this Theorem, the author used Riemann's Theorem (B. Riemann (1826–1866)) which informs us of the possibility of making the conformal representation of the region R inside the unit circle |s| = 1.

Prof. Omar Catunda addresses the following topics in his thesis: Necessary definitions such as: Complex sphere, set of points of sphere, monogeneous functions important properties, modified Cauchy formula; extension of a function, functions of two or more variables. Analytical Functional Space; Functionals of Functions of One Variable.

In this and other works by Prof. Omar Catunda, we observe an excellent mathematical background of the author, especially in Mathematical Analysis and in Analytical Functional Theory, topics in which he demonstrates solid technical skills.

Final Considerations

Our interest in this paper is to highlight only the scientific influence of Prof. Dr. L. Fantappiè on the young professors and students of the Bachelor's Degree in Mathematics at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo, during their work at that institution. He started at the University of São Paulo, the first phase of the formation process of the Brazilian mathematical community. The various phases of the formation process of the Brazilian mathematical community can be found in detail in the book (SILVA, 2023). There is no doubt that Prof. Dr. L. Fantappiè played an important role in the genesis of the Brazilian mathematical community. In this genesis, we must include Prof. Dr. Theodoro A. Ramos who, with a vision of the future and wisdom, hired Prof. L. Fantappiè to work in the Department of Mathematics at the Faculty of Philosophy, Sciences, and Letters at the University of São Paulo. This was not a decision made at random.

L. Fantappiè created it in 1935 at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo, in collaboration with Prof. Gleb Wataghin, an experimental physicist, the Mathematical and Physics Seminar at the University of São Paulo. The meetings of this Seminar were intended for the exposition, by the professors or the students, of recent research results, and also of mathematical theories that were not addressed in the programs developed in the disciplines taught. This event was a novelty for the Brazilian academic environment. In 1935 it was founded on the initiative of Prof. L. Fantappiè the Journal of Pure and Applied Mathematics of the University of São Paulo. A publication was written by Professors of the Departments of Mathematics and Physics of the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo. Its Director was Luigi Fantappiè. Volume 1, Fascicle 1, was published in June 1936. L. Fantappiè began the formation process of the mathematics library of the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo, with the donation of books made by Italian Universities and the purchase of books by the government of the state of São Paulo.

The scientific influence of Prof. Dr. L. Fantappiè together with the young professors of the Department of Mathematics at the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo was so remarkable that, after his return to Italy and, with the end of the 2nd World War, to maintain the good level of teaching and research in Mathematics at University of São Paulo, and with a vision of the future for the continuation of the process of the phases of the formation of the Brazilian mathematics community, Professors Omar Catunda and Cândido Lima da Silva Dias, as managers of the Department of Mathematics at the Faculty of Philosophia, Sciences, and Letters of University of São Paulo, hired from 1945 onwards, as Visiting Professors the mathematicians: André Weil (1906–1998), Oscar Zariski (1899–1986), Jean Dieudonné (1906–1992), Laurent Schwartz (1915–2002), Alexander Grothendieck (1928–2014), Jean F. A. Delsarte (1903–1968), Charles Ehresmann (1905–1979), and Jean-Louis Koszul (1921–2018), among others; some of which were members of the prestigious French group N. Bourbaki. This fact gave continuity to the offer of a good graduation course in Mathematics by the Faculty of Philosophy, Sciences, and Letters of the University of São Paulo. Professors Omar Catunda and Cândido Lima da Silva Dias later guided students in obtaining their doctorates in Science (Mathematics), continuing with the scientific ancestry of Prof. Luigi Fantappiè.

The example of offering good undergraduate courses in Mathematics has expanded from the University of São Paulo to other Universities in the country. The first of these was the University of Brazil in the city of Rio de Janeiro, which, in 1939, created the National Faculty of Philosophy, an institution that began to offer good undergraduate courses in Mathematics. Continuing with the first phase of the process and formation of the Brazilian mathematical community, in 1939 the Department of Mathematics of the National Faculty of Philosophy of the University of Brazil hired three Italian Prof. Achille Bassi (1907–1973) for the Chair of Geometry. He introduced in Brazil the first notions of Algebraic Topology; Prof. Gabrielle Mamanna (1893–1942) for the Chair of Mathematical Analysis. His courses in Analysis included: Ordinary Differential Equations, Calculus of Variations, Integral Equations, and Complex Functions. G. Mamanna published some articles in the Annals of the Brazilian Academy of Sciences on Calculus of Variations; Prof. Luigi Sobrero (1909–

1979), a mathematical physicist who worked on the Mathematical Theory of Linear Elasticity (Continuum Mechanics).

These Italian professors began to encourage young professors and students of the Bachelor of Mathematics course at the National Faculty of Philosophy. This institution began to offer Mathematics courses with a good level of quality. In 1965, the University of Brazil became the Federal University of Rio de Janeiro and, in this institution, the Institute of Mathematics was created. From the São Paulo - Rio de Janeiro axis, the Brazilian mathematical community flourished and spread to other states in Brazil.

Acknowledgments

We express our thanks to Prof. Dr. Luiz C. B. da Silva, from the School of Science and Engineering, University of Dundee - United Kingdom, for suggesting corrections to the structure of the paper. We also thank Prof. Dr. Gert Schubring, Universität Bielefeld, Germany, for suggesting the inclusion in the text of ideological political aspects embraced by L. Fantappiè.

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