

STOP CALLING BERNOULLI'S LAW OF LARGE NUMBERS HIS "GOLDEN THEOREM" (PLEASE?)

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Abstract

Jakob Bernoulli (1655 - 1705) proved the first form of the law of large numbers before 1690 and realized the range of applications of probability calculus would be largely widened by the result. It is pretty common to find examples in the statistical literature referring to it as his "Golden Theorem". But when did Jakob name his discovery? In fact, he never did, at least this one. A mistake in the translation of Bernoulli's major work, *Ars Conjectandi* (1713), and the fact that Bernoulli named another result as "Golden theorem" led us to propagate this mistake ... for almost 100 years.

Keywords: law of large numbers, Bernoulli's golden theorem.

[PARE DE CHAMAR A LEI DOS GRANDES NÚMEROS DE "TEOREMA DE OURO" DE BERNOULLI (POR FAVOR?)]

Resumo

Jakob Bernoulli (1655 - 1705) demonstrou a primeira versão da lei dos grandes números antes de 1690 e percebeu que o campo das aplicações do cálculo de probabilidades seria amplamente alargado pelo resultado. É bastante comum encontrar exemplos na literatura estatística que se referem a esse teorema como seu "Teorema de Ouro". Mas quando Jakob deu esse nome à sua descoberta? De fato, ele nunca o fez, pelo menos para esse teorema. Um engano cometido na tradução do principal livro de Bernoulli, *Ars Conjectandi* (1713), além do fato de ele ter nomeado outro resultado como "Teorema de Ouro", nos levaram a propagar esse engano ... por quase 100 anos.

I was recently writing a note on Jakob Bernoulli's discovery of the weak law of large numbers (LLN) when, after referring to it a few times as "Bernoulli's result", I searched for a synonym, that immediately came to me: "his Golden Theorem". Being contaminated by the scholarly virus of referencing (almost) everything, it occurred to me to mention the original text in which he named this important discovery so pompously. The task seemed quite easy since the title was, I believed, most likely given in his *Ars Conjectandi*, published posthumously in 1713 by his nephew Niklaus Bernoulli (1687 - 1759).

However, searching the English translation of Edith Sylla (Bernoulli (2006)), I could not find any mention of such a name, or any similar term assigned to the result. "Ok", I thought, "maybe Sylla chose a different term to translate golden or gold, so let me check the original Latin text".

It was not there: the original text has no reference to any precious metal whatsoever. Maybe he named it in his diary, called *Meditationes*, used by Niklaus as the main source for the book. It was not there either.¹

I also searched Jakob's correspondence with Gottfried Leibniz (1646 - 1716),² since they indeed discussed the LLN and its meaning in a series of letters. However, it was not there either. A last try would be Jakob's published papers, and there it was: an article, published in 1694 in a journal called *Acta Eruditorum*, about a curve found in Physics and known as *elastica*.³ Ok, but what was the relationship between this curve and the law of large numbers? None.

But why are there several mentions of Bernoulli's LLN as his "Golden Theorem" in the statistical literature? Just to mention two recent ones see Raper (2018) and Diaconis and Skyrms (2018), p. 64. Even historians of the field like Schneider (2001), Daston (2007) and Edwards (2013) called the discovery by that name.⁴ So, there was certainly something lost in translation, but where was it? Following the thread I have eventually found its origin: a paper published in 1925 in *Biometrika* written by Karl Pearson. At the bottom of page 206 of his article, Pearson (1925), he provided the following translation of Bernoulli (1713), page 227.

"This is therefore the problem that I now wish to publish here, having considered it closely for a period of twenty years, and it is a problem of which the novelty as well as the high utility together with its grave difficulty exceed in value all the remaining chapters of my doctrine. Before I treat of this "Golden Theorem" I will show that a few objections, which certain learned men have raised against my propositions, are not valid." (Pearson (1925), p. 206)

¹ For the *Meditationes* I searched van der Waerden (1975). See also Matmüller (2014) for more details about Bernoulli's notes on probability.

² Pulskamp (2002) and Sheynin (2020).

³ See Bernoulli (1694), p. 263. For historic details about the *elastica* see Blåsjö (2017), sec. 8.2, Levien (2008) and Peiffer (2006).

⁴ In fact, Schneider (2001) called it "theorema aureum".

The same passage was translated by Sylla as:

“This, therefore, is the problem that I have proposed to publish in this place, after I have already concealed it for twenty years. Both its novelty and its great utility combined with its equally great difficulty can add to the weight and value of all the other chapters of this theory. But before I convey its solution, let me remove a few objections that certain learned men have raised.” (Bernoulli (2006), p. 329)

Why did Pearson make such a mistake? Having attended King’s College, Cambridge, between 1875 and 1879,⁵ Pearson certainly could read Latin and Greek, so he is probably the one responsible for the translation of the excerpt. The reason for publishing such a free translation may be clarified by a photograph of the translated paragraph from the original 1713 edition of *Ars Conjectandi*.

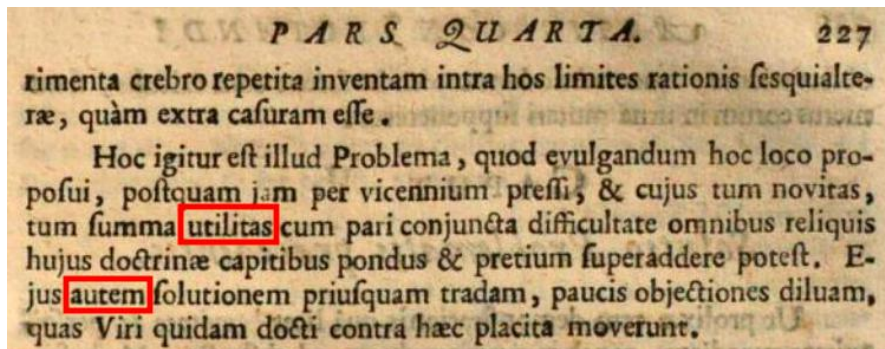


Figure 1: Top of page 227 of *Ars Conjectandi*, 1713 edition.

Pearson probably misread the “t” in “autem”, interpreting it as “r”; compare it, for instance, with the “t” from “utilitas” in Figure 1. Since “aurem” would not make any sense in the context (it is something related to “ear” or “hearing”), he considered it a typo or read it as “aureum”. Put a mistake made by an influential statistician with the fact that Jakob Bernoulli in fact named one of his discoveries as “Golden theorem” and there you go, the damage is done.

Even though my historical knowledge and audience lay miles and miles (and miles) behind those of Pearson, I can only hope the publicity offered by this journal will help us to correct this minor mistake Mr. P. has committed.

⁵ “Pearson, Carl (or Karl) (PR875CK)”. A Cambridge Alumni Database. University of Cambridge, available at <https://venn.lib.cam.ac.uk/>.

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